$Ax = b$ $\quad\quad \|Ax - b\| = \Delta$

$\Delta = 0$

min x ← mixed solution

$x^+ = A^+ b$

$\Delta = 0$

$A^+ = A^{-1}$

SINGULAR VALUE DECOMPOSITION

$A \in R^{m \times n}$

$A = P \cdot D \cdot Q^T = \underset{m}{\overset{m}{\left[\quad\right]}} \left[\begin{matrix} \sigma_1 & & & 0 \\ & \sigma_2 & & \\ & & \sigma_r & \\ 0 & & & 0 \end{matrix}\right] \underset{n}{\overset{n}{\left[\quad\right]}}$

square unitary

$\sigma_1 \neq 0$

$\sigma_2 \neq 0$

$\vdots$

$\sigma_r \neq 0$

$r = \mathrm{rank}(A)$

$P^T P = I$, $\quad P^T = P^{-1}$

$Q^T Q = I$, $\quad Q^T = Q^{-1}$

$X_N^T X_N \rightarrow [\ \ ]$

$N = 100$

$A = R^{4 \times 3}$

$A^+ = R^{3 \times 4}$

3

$Y_N = \Phi_N \Theta + Z_N$ $\quad$ $= 0$

$\| Y_N - \Phi_N \Theta \| \rightarrow \min_\Theta$

$\hat{\Theta} = \Phi_N^+ \cdot Y_N$

$\Phi_N^+ = (\Phi_N^T \Phi_N)^{-1} \Phi_N$