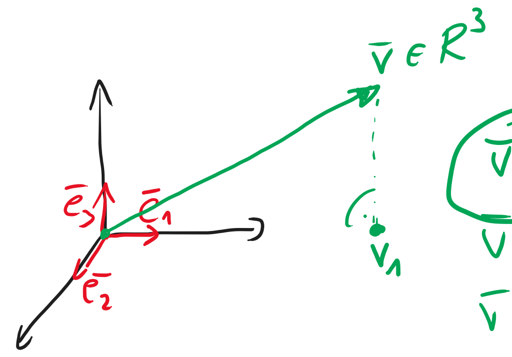


$\bar v \circ \bar e_1 = v_1$

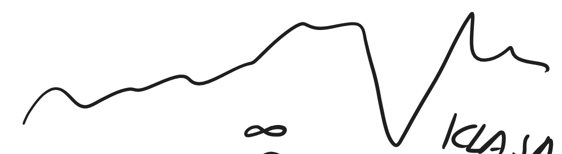
$\bar v \circ \bar e_2 = v_2$

$\bar v \circ \bar e_3 = v_3$

$\langle \bar v, \bar e_i \rangle = v_i$

$$\bar v = \underbrace{\langle \bar v, \bar e_1 \rangle}_{v_1} \bar e_1 + \underbrace{\langle \bar v, \bar e_2 \rangle}_{v_2} \bar e_2 + \underbrace{\langle \bar v, \bar e_3 \rangle}_{v_3} \bar e_3$$

$$\bar v \circ \bar v = (v_1 \bar e_1 + v_2 \bar e_2 + v_3 \bar e_3) \circ (v_1 \bar e_1 + v_2 \bar e_2 + v_3 \bar e_3)$$
$$= v_1^2 + v_2^2 + v_3^2 = \|v\|$$



$f(x) \subset L^2$ KLASA

$\int_{-\infty}^{\infty} f^2(x)dx < \infty$

wel. ot.

$\{\varphi_i(x)\}_{i=1}^{\infty}$

$$\boxed{f(x) = \sum_{i=1}^{\infty} a_i \varphi_i(x)}$$

$$\boxed{a_i = f(x) \circ \varphi_i(x)}$$

$$a_i = \int_D f(x) \cdot \varphi_i(x)\, dx = E\varphi_i(x)$$

funkcja gęstości prawdop.

POMIARY (REALIZACJE)

$x_1, x_2, x_3, \ldots, x_k, \ldots, x_N$

$$\hat a_i = \frac{1}{N} \sum_{k=1}^{N} \varphi_i(x_k)$$

$X \sim f(x)$

$\int x f(x) dx = EX$

$$\hat f(x) = \sum_{i=1}^{S} \hat a_i \cdot \varphi_i(x)$$

$\int f^2 < \infty$

L^2

$$\|f\| = f(x) \circ f(x) = \int_D (a_1\varphi_1(x) + a_2\varphi_2(x) + \ldots + a_i\varphi_i(x) + \ldots) \cdot (a_1\varphi_1(x) + a_2\varphi_2(x) + \ldots + a_i\varphi_i(x) + \ldots)\, dx$$

$$= a_1^2 + a_2^2 + \ldots a_i^2 + \ldots = \sum_{i=1}^{\infty} a_i^2 < \infty$$

RÓWNOŚĆ PARSEVALA

$a_i \to 0$, $i \to \infty$