REGRESSION FUNCTION

$E z_k = 0$
$\mathrm{var}\, z_k < \infty$
$z_k, u_k \leftarrow$ independent

$\{(u_k, y_k)\}$

$$y_k = \mu(u_k) + z_k$$

$$R(u) = E\{y_k \mid u_k = u\} = E\{\mu(u_k) + z_k \mid u_k = u\} =$$

$$= E\{\mu(u_k) \mid u_k = u\} + \underbrace{E\{z_k \mid \cancel{u_k = u}\}}_{E z_k = 0} =$$

$$= \boxed{\mu(u)}$$

HOW TO ESTIMATE $\mu(u)$?

$g = \mu \cdot f$

p.d.f. of u_k : $f(u)$

$$\mu(u) = \frac{\mu(u)\cdot \boxed{f(u)}}{f(u)} = \frac{g(u)}{f(u)} = \frac{\sum_{i=1}^{\infty} b_i \varphi_i(u)}{\sum_{i=1}^{\infty} a_i \varphi_i(u)}$$

$b_i = E\, y_k \varphi_i(u_k)$

$a_i = E\, \varphi_i(u_k)$

$$\hat{\mu}(u) = \frac{\cancel{\frac{1}{Nh}} \sum_{k=1}^{N} y_k K\left(\frac{u_k - u}{h}\right)}{\cancel{\frac{1}{Nh}} \sum_{k=1}^{N} K\left(\frac{u_k - u}{h}\right)}$$

$\{(u_k, y_k)\}_{k=1}^{N}$

$$\boxed{\hat{\mu}(u) = \frac{\sum_{k=1}^{N} y_k \cdot K\left(\frac{u_k - u}{h}\right)}{\sum_{k=1}^{N} K\left(\frac{u_k - u}{h}\right)}}$$

$$\hat{\theta} = \frac{\sum \theta_k \cdot w_k}{\sum w_k}$$

$$\boxed{\hat{\mu}(u) = \frac{\sum_{i=1}^{S} \hat{b}_i \varphi_i(u)}{\sum_{i=1}^{S} \hat{a}_i \varphi_i(u)}}$$

$$\hat{b}_i = \frac{1}{N} \sum_{k=1}^{N} y_k \varphi_i(u_k)$$

$$\hat{a}_i = \frac{1}{N} \sum_{k=1}^{N} \varphi_i(u_k)$$