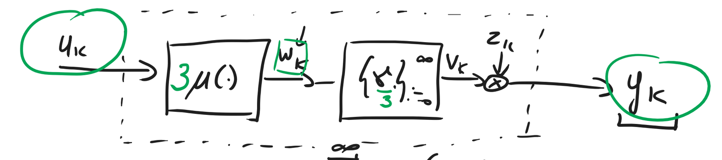
$E w_k = \bar{w}$

$\{(u_k, y_k)\}_{k=1}^{N}$

$E\{z_k \mid u_k = u\} = E\{z_k\} = 0$



$u_k \sim$ i.i.d.

$E z_k = 0 \qquad \operatorname{var} z_k < \infty$

z_k, u_k – niezależne

$$y_k = \sum_{i=0}^{\infty} \gamma_i \mu(u_{k-i}) + z_k$$

$$R(u) \triangleq E\{y_k \mid u_k = u\} = E\{\gamma_0 \mu(u_k) + \underbrace{\sum_{i=1}^{\infty} \gamma_i \overbrace{\mu(u_{k-i})}^{w_{k-i}}}_{c_1} + \cancel{z_k} \mid u_k = u\} = \gamma_0 \mu(u) + c_1$$

$$c_1 = \bar{w} \cdot \sum_{i=1}^{\infty} \gamma_i$$

$$R(u, v) \triangleq E\{y_k \mid u_k = u \wedge u_{k-1} = v\} = E\left\{\gamma_0 \mu(u_k) + \gamma_1 \mu(u_{k-1}) + \sum_{i=2}^{\infty} \gamma_i w_{k-i} + \cancel{z_k} \,\middle|\, \begin{matrix} u_k = u \\ u_{k-1} = v \end{matrix}\right\} =$$

$$= \gamma_0 \mu(u) + \gamma_1 \mu(v) + c_2$$

$$c_2 = \bar{w} \cdot \sum_{i=2}^{\infty} \gamma_i$$

$\vdots$

$$R(\underbrace{u^{(0)}, u^{(1)}, \ldots, u^{(s-1)}}_{s\text{-wym}}) = \gamma_0 \mu(u^{(0)}) + \gamma_1 \mu(u^{(1)}) + \ldots + \gamma_{s-1} \mu(u^{(s-1)}) + c_s$$

$$c_s = \bar{w} \cdot \sum_{i=s}^{\infty} \gamma_i$$

NIECH BLOK LINIOWY – FIR(s)

DLA $i \geq s \rightarrow \gamma_i = 0$

$= 0$

MODEL PARAMETRYCZNY $\mu(\cdot)$

$f_1(), \ldots f_m() \leftarrow$ znane bazowe

$$\mu(u) = c_1 f_1(u) + c_2 f_2(u) + \ldots + c_m f_m(u)$$

(H)

$$y_k = \gamma_0 (c_1 f_1(u_k) + \ldots + c_m f_m(u_k)) + \gamma_1 (c_1 f_1(u_{k-1}) + \ldots + c_m f_m(u_{k-1})) + \ldots + \gamma_{s-1} (c_1 f_1(u_{k-(s-1)}) + \ldots c_m) + z_k =$$

$$= \underbrace{\begin{bmatrix} \gamma_0 c_1 \\ \gamma_0 c_2 \\ \vdots \\ \gamma_0 c_m \\ \gamma_1 c_1 \\ \gamma_1 c_1 \\ \vdots \\ \gamma_1 c_m \\ \vdots \\ \gamma_{s-1} c_1 \\ \vdots \\ \gamma_{s-1} c_m \end{bmatrix}^T}_{\Theta} \cdot \underbrace{\begin{bmatrix} f_1(u_k) \\ \vdots \\ f_m(u_k) \\ f_1(u_{k-1}) \\ \vdots \\ f_m(u_{k-1}) \\ \vdots \\ f_1(u_{k-(s-1)}) \\ \vdots \\ f_m(u_{k-(s-1)}) \end{bmatrix}}_{\phi_k} + z_k = \Theta^T \cdot \phi_k + z_k$$

$$Y_N = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} \qquad \Phi_N = \begin{bmatrix} \phi_1^T \\ \vdots \\ \phi_N^T \end{bmatrix}$$

$$\hat{\Theta} = (\Phi_N^T \Phi_N)^{-1} \Phi_N^T Y_N$$

Bai, 1998

$$A = m\Bigg\{\underbrace{\begin{bmatrix} \gamma_0 c_1, & \gamma_1 c_1, & \ldots, & \gamma_{s-1} c_1 \\ \vdots & \vdots & & \\ \gamma_0 c_m, & \gamma_1 c_m, & \ldots, & \gamma_{s-1} c_m \end{bmatrix}}_{s} = \begin{bmatrix} c_1 \\ \vdots \\ c_m \end{bmatrix} \cdot [\gamma_0, \ldots, \gamma_{s-1}]$$

$$A = PDQ^T$$