NONPARAMETRIC APPROACH TO IDENTIFICATION OF BLOCK-ORIENTED SYSTEMS

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Statement of the problem

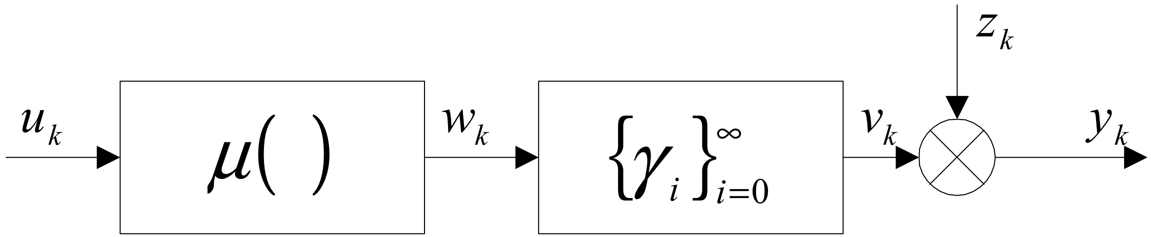


Fig.1. Hammerstein system

$$y_k = \sum_{i=0}^{\infty} \gamma_i \mu(u_{k-i}) + z_k$$

$$R(u) = \mathbf{E}\{y_k \mid u_k = u\} = c\mu(u) + s$$

NONPARAMETRIC METHODS

Kernel regression estimation

$$\hat{\mu}_M(u) = \sum_{i=1}^{M} y_i K\left(\frac{u-u_i}{h(M)}\right) \Big/ \sum_{i=1}^{M} K\left(\frac{u-u_i}{h(M)}\right)$$

Theorem 1. If $h(M) \to 0$ and $Mh(M) \to \infty$ as $M \to \infty$ then

$$\hat{\mu}_M(u) \to \mu(u)$$

in probability as $M \to \infty$ in each continuity point of $\mu()$ and the input probability density.

Theorem 2. If $\mu()$ is twice differentiable in the point u then for $h(M) = O(M^{-1/5})$ it holds that

$$\left| \hat{\mu}_M(u) - \mu(u) \right| = O(M^{-2/5})$$

Orthogonal expansion

$$\mu(u) = \frac{g(u)}{f(u)}, \text{ where } g(u) = \mu(u) f(u)$$

$$g(u) = \sum_{i=0}^{\infty} a_i \phi_i(u), \quad f(u) = \sum_{i=0}^{\infty} b_i \phi_i(u)$$

$$a_i = \mathbf{E} y_k \phi_i(u_k), \quad b_i = \mathbf{E} \phi_i(u_k)$$

$$\hat{a}_i = \frac{1}{M} \sum_{k=1}^{M} y_k \phi_i(u_k), \quad \hat{b}_i = \frac{1}{M} \sum_{k=1}^{M} \phi_i(u_k)$$

$$\hat{\mu}_M(u) = \frac{\sum_{i=0}^{q(M)} \hat{a}_i \phi_i(u)}{\sum_{i=0}^{q(M)} \hat{b}_i \phi_i(u)}$$

The convergence conditions:

trigonometric series	$\lim_{M\to\infty} \frac{q^2(M)}{M} = 0$
Legendre series	$\lim_{M\to\infty} \frac{q^2(M)}{M} = 0$
Laguerre series	$\lim_{M\to\infty} \frac{q^6(M)}{M} = 0$
Hermite series	$\lim_{M\to\infty} \frac{q^{5/3}(M)}{M} = 0$
Daubechies wavelets	$\lim_{M\to\infty} \frac{2^{2q(M)+2}}{M} = 0$

COMBINED PARAMETRIC-NONPARAMETRIC ALGORITHMS

Estimation of the static nonlinearity

$$\mu(u_k) = \mu(u_k, c) \qquad \mu(u_k) = e^{c_1 u} + \sin(c_2 u_k)$$

$$\hat{c}_{N,M} = \arg\min_c \sum_{k=1}^{N} \left(\hat{w}_{k,M} - \mu(u_k, c) \right)^2$$

$$\hat{c}_{N,M} = \left(\Phi_N^T \Phi_N \right)^{-1} \Phi_N^T \hat{W}_{N,M}$$

$$\Phi_N = (\phi_1(u_k), \phi_2(u_k), \ldots, \phi_N(u_k))^T$$

$$\phi(u_k) = \left(f_1(u_k), f_2(u_k), \ldots, f_m(u_k) \right)^T$$

Identification of the linear dynamics

$$y_k = \vartheta_k^T \theta + z_k$$

$$\theta = (\alpha_0, \alpha_1, ..., \alpha_s, \beta_1, \beta_2, ..., \beta_p)^T$$

$$\vartheta_k = (w_k, w_{k-1}, ..., w_{k-s}, y_{k-1}, y_{k-2}, ..., y_{k-p})^T$$

$$\hat{\theta}_{N,M} = \left(\hat{\Psi}_{N,M}^T \hat{\Theta}_{N,M} \right)^{-1} \hat{\Psi}_{N,M}^T Y_N$$

$$Y_N = (y_1, y_2, ..., y_N)^T$$

$$\hat{\Theta}_{N,M} = (\vartheta_1, \vartheta_2, ..., \vartheta_N)^T$$

Nonparametric instrumental variables

$$\psi_k^* = (w_k, \ldots, w_{k-s}, \bar{y}_{k-1}, \ldots, \bar{y}_{k-p})^T \qquad \psi_k^{\#} = (w_k, \ldots, w_{k-s}, \tilde{y}_{k-1}, \ldots, \tilde{y}_{k-p})^T$$

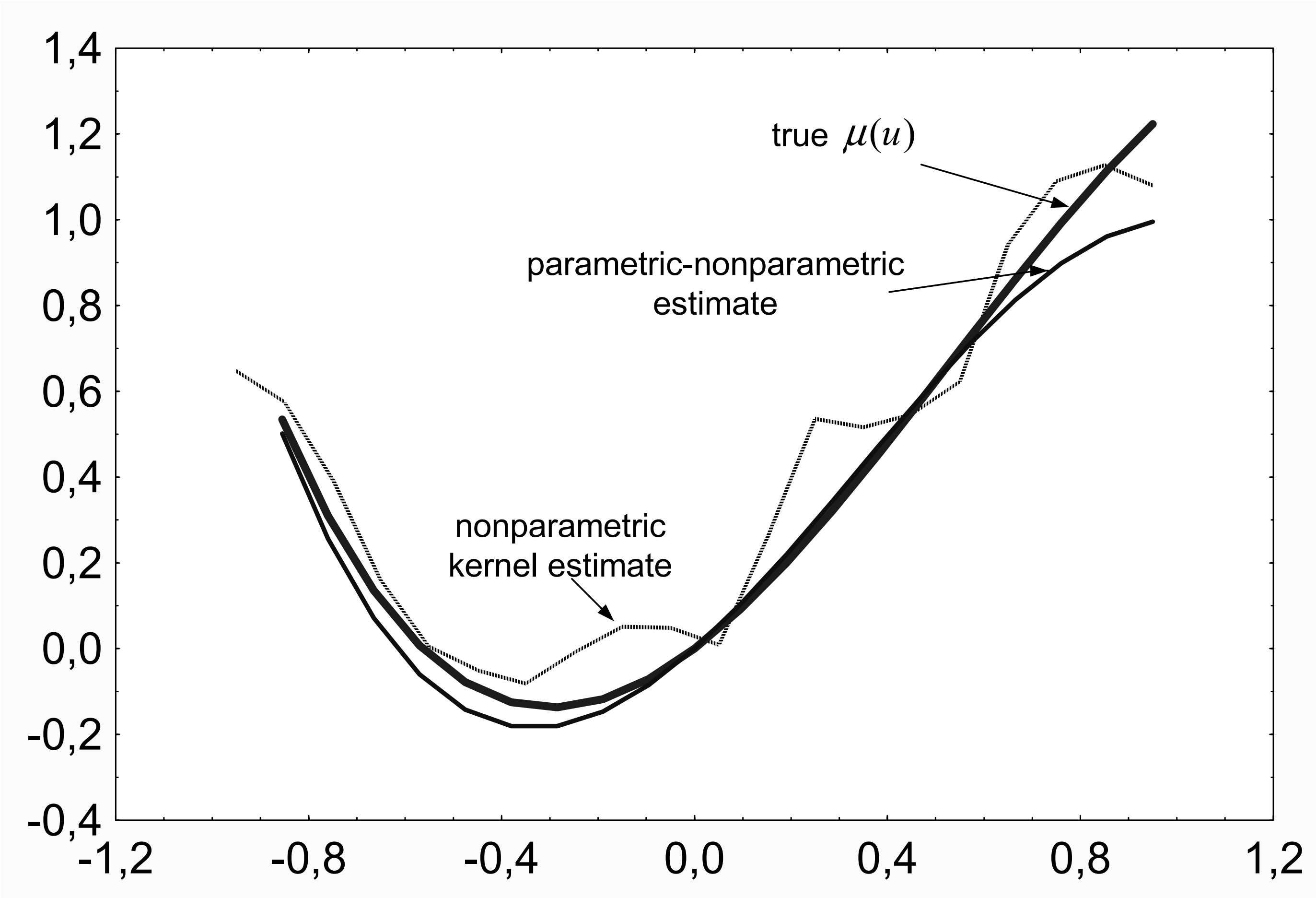
$$\tilde{y}_k = \sum_{i=0}^{APR} \hat{\gamma}_{i,M} \hat{w}_{k-i,M}$$

$$\hat{\gamma}_{i,M} = \hat{\chi}_{i,M} / \hat{\chi}_{0,M}$$

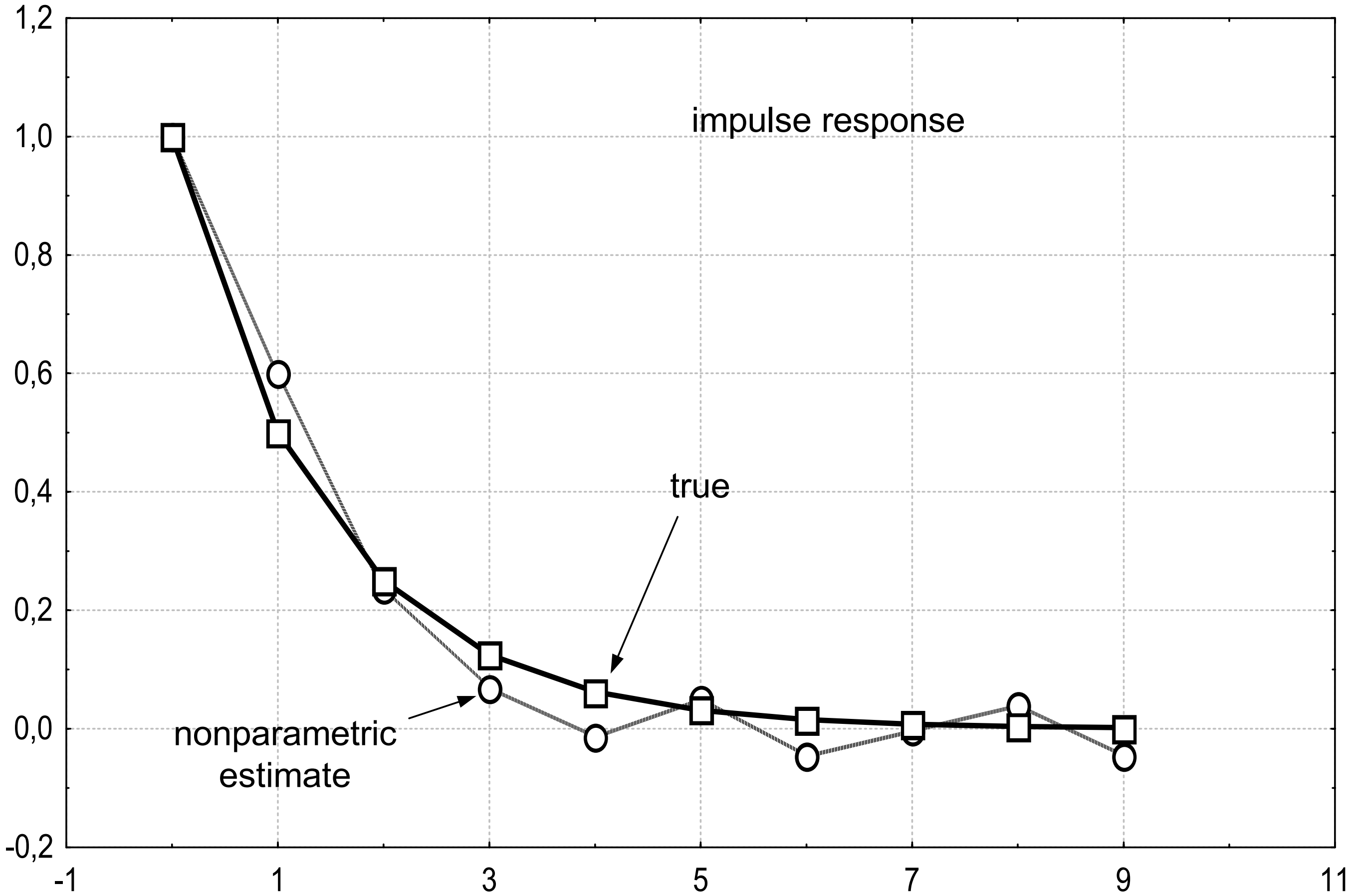
$$\hat{\chi}_{i,M} = \frac{1}{M} \sum_{k=1}^{M-i} (y_{k+i} - \bar{y})(u_k - \bar{u})$$

$$\bar{y} = \frac{1}{M} \sum_{k=1}^{M} y_k, \quad \bar{u} = \frac{1}{M} \sum_{k=1}^{M} u_k$$

Example



true $\mu(u)$
parametric-nonparametric estimate
nonparametric kernel estimate
1,4
1,2
1,0
0,8
0,6
0,4
0,2
0,0
-0,2
-0,4
-1,2
-0,8
-0,4
0,0
0,4
0,8
1,2


impulse response
true
nonparametric
estimate
1,2
1,0
0,8
0,6
0,4
0,2
0,0
-0,2
-1
1
3
5
7
9
11

Conclusions

- Each part is identified separately
- The convergence is strictly proved

  Hasiewicz, Z. and G. Mzyk (2004). Combined parametric-nonparametric identification of Hammerstein systems. *IEEE Transactions on Automatic Control*, vol. 49, pp. 1370-1376.

  Hasiewicz, Z. and G. Mzyk (2006). Nonparametric instrumental variables for Hammerstein system identification. *IEEE Transactions on Automatic Control* (submitted to)
- The method works under existence of correlated random noise