Multi-level control in complex systems – identification and optimization

Grzegorz Mzyk

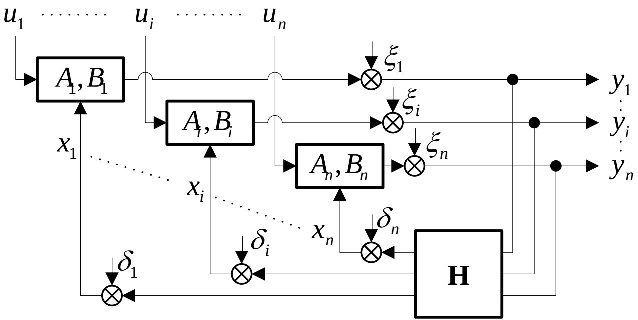
Wrocław Univ. of. Sci. and Tech., W04N

2021

Schedule

1. Statement of the problem (identification and control)
2. Identifiability conditions
3. Complex ssystem identification
4. Two-level optimization of control
5. Decomposition and coordination
6. Summary, literature

System description



Complex system

System description

$$y_i = A_i x_i + B_i u_i + \xi_i \qquad (i = 1, 2, ..., n),$$

$$\begin{aligned} u &= (u_1, u_2, ..., u_n)^T \\ x &= (x_1, x_2, ..., x_n)^T \\ y &= (y_1, y_2, ..., y_n)^T \end{aligned}$$

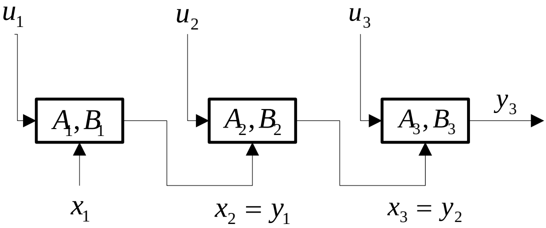
$$x_i = H_i y + \delta_i$$

System description

$$A = \mathrm{diag}(A_1, A_2, ..., A_n)$$
$$B = \mathrm{diag}(B_1, B_2, ..., B_n)$$
$$H = \left(H_1^T, H_2^T, ..., H_n^T\right)^T$$

$$\begin{cases} y = Ax + Bu + \xi \\ x = Hy + \delta \end{cases}$$

Example – cascade system



Cascade system

$$H = \begin{bmatrix} 0 & 0 & 0 \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}$$

Identifiability

Cascade system

Twierdzenie

I-th element is identifiable in cascade system if and only if the following condition holds

$$rank\left[A_{i-1}A_{i-2}...A_1,\ A_{i-1}A_{i-2}...B_1,\ ...,\ A_{i-1}B_{i-2},\ B_{i-1}\right] = \dim x_i$$

[Hasiewicz, *Int. J. Sys. Sci.*, 1987]

Identifiablility

General structure

Twierdzenie

I-th element is identifiable in general system if and only if the following condition holds

$$rankH_i\left[K^{(1)},...,K^{(i-1)},K^{(i+1)},...,K^{(n)}\right]=\dim x_i,$$

where $K^{(i)}$ *denotes i-th column block of the matrix*

$$K=(I-AH)^{-1}B.$$

[Hasiewicz, *Int. J. Sys. Sci.*, 1987]

Least squares

$$
\begin{aligned}
Y_{iN} &= [y_i^{(1)}, y_i^{(2)}, \dots, y_i^{(N)}] \\
W_{iN} &= [w_i^{(1)}, w_i^{(2)}, \dots, w_i^{(N)}] \\
w_i &= (x_i, u_i)^T
\end{aligned}
$$

$$Y_{iN} = (A_i, B_i) W_{iN} + \xi_i$$

$$\widetilde{w}_i = (\widetilde{x}_i, u_i)^T, \qquad \widetilde{x}_i = H_i y = x_i - \delta_i$$

$$\widetilde{W}_{iN} = [\widetilde{w}_i^{(1)}, \widetilde{w}_i^{(2)}, \dots, \widetilde{w}_i^{(N)}]$$

$$(\widehat{A}_i^{l.s.}, \widehat{B}_i^{l.s.}) = Y_{iN} \widetilde{W}_{iN}^T \left(\widetilde{W}_{iN} \widetilde{W}_{iN}^T \right)^{-1}$$

Instrumental variables

$$
\begin{aligned}
(\widehat{A}_i^{l.s.}, \widehat{B}_i^{l.s.}) &= Y_{iN} \widetilde{W}_{iN}^T \left(\widetilde{W}_{iN} \widetilde{W}_{iN}^T \right)^{-1} \\
(\widehat{A}_i^{i.v.}, \widehat{B}_i^{i.v.}) &= Y_{iN} \Psi_{iN}^T \left(\widetilde{W}_{iN}^T \Psi_{iN}^T \right)^{-1},
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{W}_{iN} &= [\widetilde{w}_i^{(1)}, \widetilde{w}_i^{(2)}, \ldots, \widetilde{w}_i^{(N)}] \\
\Psi_{iN} &= [\psi_i^{(1)}, \psi_i^{(2)}, \ldots, \psi_i^{(N)}]
\end{aligned}
$$

$$
\psi_i^{(k)} = \left(\psi_{i,1}^{(k)}, \psi_{i,2}^{(k)} \right)^T .
$$

optimal instruments

$$
\psi_i^* = \overline{w}_i = (\overline{x}_i, u_i)^T, \qquad \overline{x}_i = E(x_i | u) = H_i K u
$$

Global approach

$$\begin{cases} y = Ax + Bu + \xi \\ x = Hy + \delta \end{cases}$$

$$\begin{aligned} y &= A\left(Hy + \delta\right) + Bu + \xi, \\ \left(I - AH\right) y &= Bu + A\delta + \xi, \end{aligned}$$

$$y = Ku + G\theta$$

$$K = \left(I - AH\right)^{-1} B$$

control

$$u = K^{-1} y_{\dot{z}}$$

Local contrlo

local decision u_i (balancing)

$$y_{\dot{z},i} = a_i x_i + b_i u_i$$

$$x_i = H_i y_{\dot{z}}$$

control

$$u_i = \frac{y_{\dot{z},i} - a_i H_i y_{\dot{z}}}{b_i}$$

Objective function and constraints

constraints

$$
\begin{aligned}
(u_i, x_i) &\in C_i \\
\sum_i r_i(u_i, x_i) &\leq r_0
\end{aligned}
$$

local quality/performance index

$$Q_i(u_i, x_i)$$

global index

$$
\begin{aligned}
Q &= \psi(Q_1, Q_2, \ldots, Q_n) \\
& \quad \psi() \text{ -- order preserving function} \\
\text{e.g. } Q &= \sum_i Q_i
\end{aligned}
$$

Two-level optimization (with decomposition)

$$u = \left(u^{(1)}, u^{(2)}\right)$$

$u^{(1)}$ – upper level decisions (coordination variables)

$u^{(2)}$ – lower level controls

$$\max_{u} Q(u) = \max_{u^{(1)}} \left\{ \max_{u^{(2)}} Q\left(u^{(1)}, u^{(2)}\right) \right\}$$

Coordination procedures
Direct method

upper level

$$\begin{aligned} Q &= \psi\left(Q_1(y_d, r_{d_1}), \ldots, Q_n(y_d, r_{d_n})\right) \rightarrow \max_{y_d, r_d} \\ \sum_i r_{d_i} &\leq r_0 \end{aligned}$$

lower level

$$\begin{aligned} Q_i(u_i, x_i) &\rightarrow \max_{u_i} \\ x_i &= H_i y_d, \qquad (u_i, x_i) \in C_i, \qquad r_i \leq r_{d_i} \end{aligned}$$

problem: for any (y_d, r_d), the solution may not exist

$$(y_d, r_d) \in YR$$

set YR – difficult to determine (depends on system structure and contstraints)

Coordination procedures
Penalty method

upper level

$$\overline{Q} = \psi\left(\overline{Q}_1(y_d, r_{d_1}), \ldots, \overline{Q}_n(y_d, r_{d_n})\right) \longrightarrow \max_{y_d, r_d}$$

lower level

$$\overline{Q}_i = Q_i(u_i, x_i) - K(y_i - y_{d_i}) \longrightarrow \max_{u_i}$$

Coordination procedures

Price method

upper level

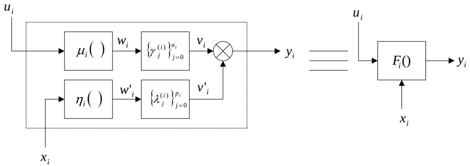
$$\phi(\lambda) = \sum_i Q_i \left(u_i(\lambda), x_i(\lambda)\right) + \langle \lambda, x(\lambda) - Hy(\lambda) \rangle \to \min_{\lambda}$$

lower level

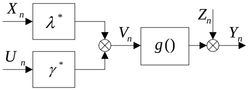
$$\widetilde{Q} = Q_i(u_i, x_i) + \langle \lambda_i, u_i \rangle - \langle \mu_i, y_i \rangle \to \min_{u_i}$$

Generalization

Nonlinear systems



Hammerstein-type block



Wiener-type block

Literature

[1] Findeisen, W., Bailey, F.N., Brdyś, M., Malinowski, K., Tatjewski, P.,Woźniak, A.: *Control and Coordination in Hierarchical Systems*. J. Wiley, Chichester (1980)

[2] Findeisen, W., *Wielopoziomowe układy sterowania*, PWN, Warszawa (1974)

M. D. Mesarowic, D. Macko, Y. Takahara, *Theory of Hierarchical, Multilevel Systems*, Academic Press, New York, 1970.