

# Making decisions under conditions of uncertainty

*Grzegorz Mzyk, Assoc. Prof.*

*Wrocław University of Science and Technology*



HR EXCELLENCE IN RESEARCH



Politechnika Wrocławska

# Examples

- Waiting time in the queue
- chicken egg mass
- Currency rate
- No. of car accidents
- number of current calls in the telephone exchange
- the patient's response to the dose of the drug
- weather forecast
- time of failure-free operation of the device

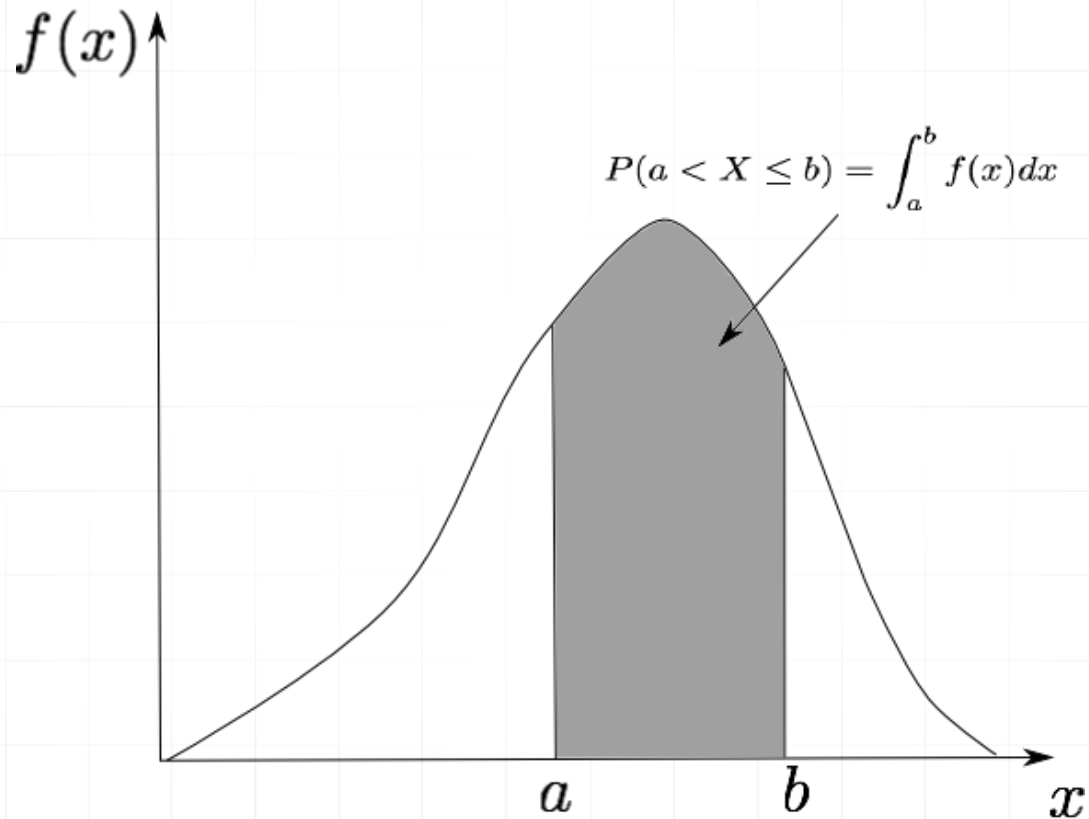
# Random variable, $X$

Random event  $\rightarrow$  Real number

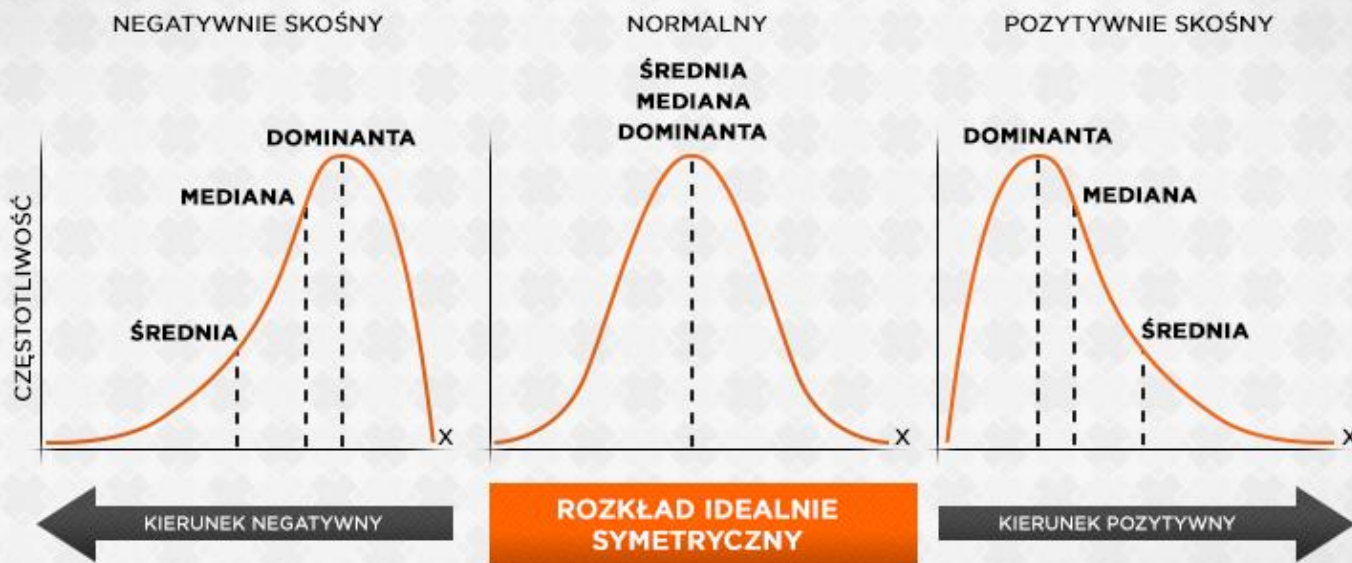
„eagle”  $\rightarrow X=1$

„tails”  $\rightarrow X=0$

# Probability density function



# Expected mean (EX), median and dominant



źródło: [www.pinnacle.com](http://www.pinnacle.com)

$$EX = \int x f(x) dx$$

# Decision, $D$

Make decision  $D$ , to be „**as close as possible**” to  $X$

How to assess the quality of the decision?

$D - X$  is not a measure

# Loss function, $L$

Examples:

$$L(D, X) = |D - X|$$

$$L(D, X) = (D - X)^2$$

$$L(D, X) = -\delta(D - X)$$

Note:  $L$  is random

# Risk, $R$

$$R(D) = E\{L(D, X)\}$$

Operator  $E\{ \ }$   
averages all possible  $X$ ,  
 $R( \ )$  depends on  $D$  only and is **not** random



# Optimal decision

$$D_{opt} = \underset{D}{\operatorname{argmin}} R(D)$$

$D_{opt}$  depends on  $L$

# Typical cases

$L$	
$ D - X $	Median $X$
$(D - X)^2$	$EX$ (mean)
$-\delta (D - X)$	Dominant $X$

# Example

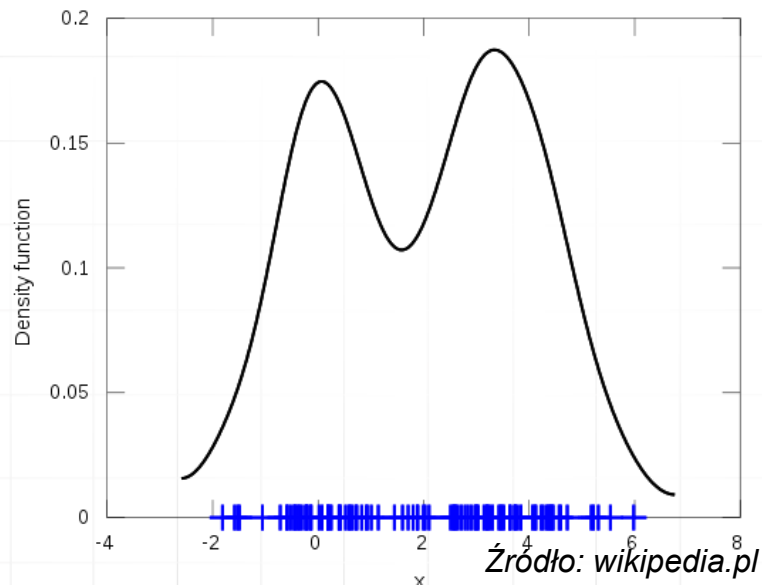
$$X = \begin{cases} 1, & \text{with probability } 1/3 \\ 2, & \text{with probability } 1/3 \\ 6, & \text{with probability } 1/3 \end{cases}$$

$L$	$D_{opt}$
$ D - X $	2
$(D - X)^2$	3
$-\delta (D - X)$	1, 2, or 6

# Learning

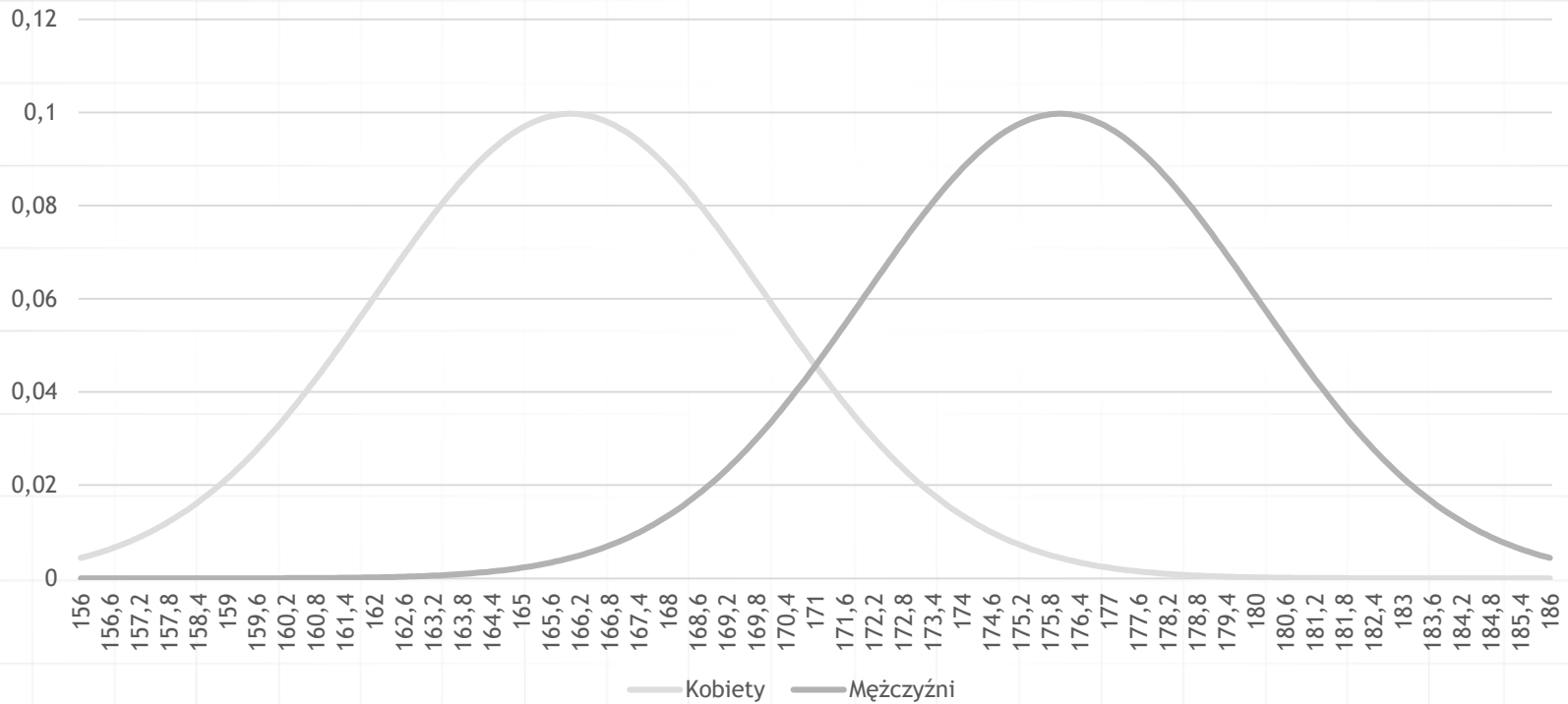
To make optimal decision you must know  $f(x)$

$$x_1, x_2, \dots, x_N \rightarrow \hat{f}(x)$$



# Recognition

Body height for men and women



# Summary

- is there justice in the world?
- do security systems improve security?
- is the data analysis ethically correct?
- how to aggregate decisions of many entities?
- how many features to take into account? (dimensions)