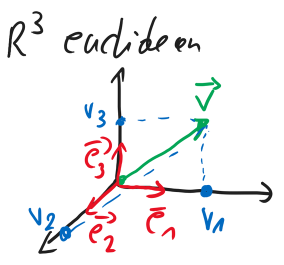
R^3 euclidean



$i \neq j \quad \vec{e}_i \circ \vec{e}_j = 0 \;, \quad i = j \quad \vec{e}_i \circ \vec{e}_j = 1$

$\|\vec{e}_i\| = 1$

$v_1 = \vec{v} \circ \vec{e}_1 = \langle \vec{v}, \vec{e}_1 \rangle$

$v_2 = \vec{v} \circ \vec{e}_2 = \langle \vec{v}, \vec{e}_2 \rangle$

$v_3 = \vec{v} \circ \vec{e}_3 = \langle \vec{v}, \vec{e}_3 \rangle$

$\vec{v} = (v_1, v_2, v_3) = v_1 \cdot \vec{e}_1 + v_2 \cdot \vec{e}_2 + v_3 \cdot \vec{e}_3 =$

$= \langle \bar{v}, \bar{e}_1 \rangle \vec{e}_1 + \langle \bar{v}, \bar{e}_2 \rangle \bar{e}_2 + \langle \bar{v}, \bar{e}_3 \rangle \cdot \vec{e}_3$

$\vec{v} \circ \vec{v} = (v_1 \bar{e}_1 + v_2 \bar{e}_2 + v_3 \bar{e}_3) \circ$

$\circ (v_1 \bar{e}_1 + v_2 e_2 + v_3 e_3) = \underbrace{v_1^2 + v_2^2 + v_3^3}_{\text{NORM.}}$

$f(x) \in L_2$ class of functions

WE ASSUME THAT $\int f^2(x) dx < \infty$

I HAVE! GIVEN (WIKI) p.d.f

$\{\varphi_i(x)\}_{i=1}^{\infty}$

$$f(x) = \sum_{i=1}^{\infty} a_i \varphi_i(x),$$

where $a_i = f(x) \circ \varphi_i(x)$

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if $f(x)$ denotes prob. den. fu.

$$a_i = \int f(x) \varphi_i(x) dx = E\varphi_i(x)$$

$x_1, x_2, x_3, \ldots\ldots, x_k, \ldots\ldots, x_N$

$$\hat{a}_i = \frac{1}{N} \sum_{k=1}^{N} \varphi_i(x_k)$$

$i = 1, 2, 3, 4, \ldots$

$k = 1, \ldots, N$

$$\hat{f}(x) = \sum_{i=1}^{S} \hat{a}_i \varphi_i(x)$$

S-scale

ANY $x \in \mathbb{R}$

$$\|f(x)\| = f(x) \circ f(x) \stackrel{\text{def}}{=} \underbrace{\int_D f(x) \cdot f(x)\, dx}_{\int f^2} = \int_D (a_1\varphi_1(x) + a_2\varphi_2(x) + \ldots)(a_1\varphi_1(x) + a_2\varphi_2(x) + \ldots) dx =$$

$$= a_1^2 + a_2^2 + a_3^2 + \ldots = \sum_{i=1}^{\infty} a_i^2 < \infty$$

PARSEVAL EQALITY

$\hat{a}_i \xrightarrow{p.1} a_i$ for each individual $i = 1, 2, 3, \ldots$

CONSISTENCY CONDITIONS:

$S(N) \to \infty$ as $N \to \infty$

$\frac{S(N)}{N} \to 0$

$\Rightarrow \quad \hat{f}(x) \xrightarrow[N \to \infty]{p.1} f(x)$

e.g. $S = \sqrt{N}$