LINEAR SYSTEM

$$\mathbf{x} = \begin{bmatrix} x^{(1)} \\ \vdots \\ x^{(s)} \end{bmatrix} \qquad a^* = \begin{bmatrix} a^{*(1)} \\ \vdots \\ a^{*(s)} \end{bmatrix}$$

$$y = \underbrace{\mathbf{x}^T a^* + z = \underbrace{a^{*T}\mathbf{x}}_{\text{scalar product}} + z}$$

MEASUREMENT DATA

$k = 1, 2, \ldots, N$

$$y_k = \mathbf{x}_k^T a^* + z_k$$

N equations

DATABASES:
INFORMATION MATRICES

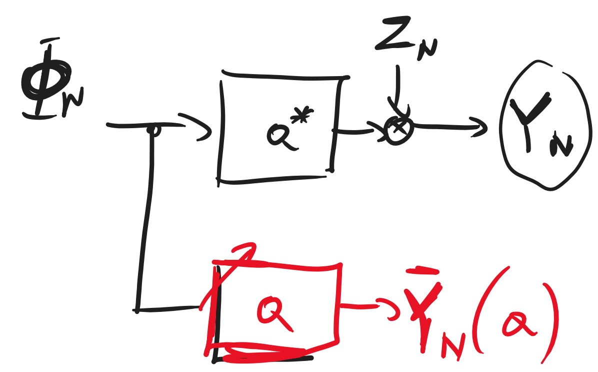
UNKNOWN

$$Y_N = \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}}_{1} \Big\} N \qquad \Phi_N = \underbrace{\begin{bmatrix} x_1^{(1)} & \ldots & x_1^{(s)} \\ x_2^{(1)} & \ldots & x_2^{(s)} \\ \vdots & & \\ x_N^{(1)} & \ldots & x_N^{(s)} \end{bmatrix}}_{s} = \begin{bmatrix} \mathbf{x}_1^T \\ \mathbf{x}_2^T \\ \vdots \\ \mathbf{x}_N^T \end{bmatrix} \qquad Z_N = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_N \end{bmatrix}$$

$E z_k = 0$

$$Y_N = \Phi_N \cdot a^* + Z_N$$

$$\bar{Y}_N(a) = \Phi_N a$$



$$\| Y_N - Y_N(a) \| = (Y_N - Y_N(a))^T \cdot (Y_N - Y_N(a)) =$$

$$\|v\| = v \circ v = v^T \cdot v$$

NORM OF VECTOR

$$= (Y_N - \Phi_N a)^T (Y_N - \Phi_N a) = Q(a)$$

$$\nabla_a Q(a) = 0$$

CONSEQUENCE:

$$\Phi_N^T \Phi_N a = \Phi_N^T Y_N$$

NORMAL EQUATION

OPTIMAL

$$\hat{a} = (\Phi_N^T \Phi_N)^{-1} \Phi_N^T Y_N$$

LEAST SQUARES ESTIMATE

$$\Phi_N^T \cdot (\Phi_N a - Y_N) = 0$$