$$Y_N = \Phi_N a^* + Z_N$$

$$\Phi_N = \begin{bmatrix} \phi_1^T \\ \phi_2^T \\ \vdots \\ \phi_{k}^T \\ \vdots \\ \phi_N^T \end{bmatrix} = \begin{bmatrix} \\ \\ \\ u_k, u_{k-1}, \dots u_{k-s}, y_{k-1}, \dots y_{k-p} \\ \\ \end{bmatrix}$$

$$y_k = \phi_k^T a^* + z_k, \quad k = 1, \dots, N$$

$a^* \in$ TRUE OBJECT IS STABLE $\Rightarrow$ OUTPUT IS ERGODIC

LS:

$$\hat{a} = \left(\Phi_N^T \Phi_N\right)^{-1} \Phi_N^T Y_N = \left(\frac{1}{N}\sum_{k=1}^{N} \phi_k \phi_k^T\right)^{-1} \left(\frac{1}{N}\sum_{k=1}^{N} \phi_k y_k\right) = \left(\frac{1}{N}\sum \phi_k \phi_k^T\right)^{-1} \frac{1}{N}\sum \phi_k \phi_k^T \cdot a^* + \text{Error}$$

SLLN

$$\frac{1}{N}\sum \phi_k \phi_k^T \xrightarrow{\text{P.1.}} E\,\phi_k \phi_k^T = E\,\phi_1 \phi_1^T$$

$$\text{Error} = \left(\frac{1}{N}\sum_{k=1}^{N} \phi_k \phi_k^T\right)^{-1} \frac{1}{N}\sum_{k=1}^{N} \phi_k z_k$$

$$\frac{1}{N}\sum_{k=1}^{N} \phi_k z_k \xrightarrow{\text{P.1}} E\,\phi_k z_k = E \begin{bmatrix} u_k \\ u_{k-1} \\ \vdots \\ u_{k-s} \\ y_{k-1} \\ \vdots \\ y_{k-p} \end{bmatrix} \cdot z_k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ \neq 0 \\ \neq 0 \\ \vdots \\ \neq 0 \end{bmatrix}$$

u, z - indep.

$$0 = E\,\phi_k z_k = E\,\phi_k \cdot E\,z_k$$

$E\,z_k \neq 0$