Hybrid Identification of L-N-L and N-L-N Systems

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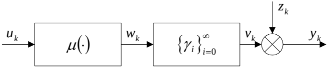
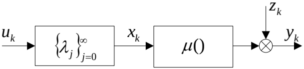
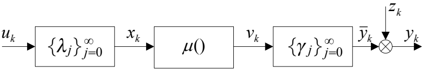
Block-oriented systems



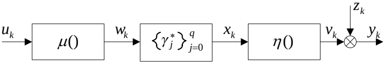
Hammerstein system (N-L)



Wiener system (L-N)



Wiener-Hammerstein system (L-N-L)



Hammerstein-Wiener system (N-L-N)

Parametric vs. nonparaetric approach

Parametric

the system can be described with the use of **finite and known** number of parameters

Non-parametric

the class of transformation is more general, the number of necessary parameters is **infinite or unknown**

Examples of parametric models

nonlinear static block with polynomial characteristics

$$\mu(u) = \theta_p u^p + \theta_{p-1} u^{p-1} + \ldots + \theta_1 u + \theta_0$$

p – known and finite
θ_i – unknown $(i = 0, 1, \ldots, p)$

linear dynamics with ARMA difference equation

$$y_k = a_1 y_{k-1} + \ldots a_m y_{k-m} + b_0 u_k + \ldots + b_n u_{k-n}$$

$m,\ n$ – known and finit
$a_i,\ b_j$ – unknown $(i = 1, 2, \ldots, p,\ \ j = 0, 1, \ldots, n)$

Typical assumptions in nonparametric approach

About nonlinearity

square integrable

Lipschitz

continuous

differentiable

invertible

About linear dynamics

asymptotically stable

invertible

About excitations

i.i.d. (normal/uniform distribution)

bounded

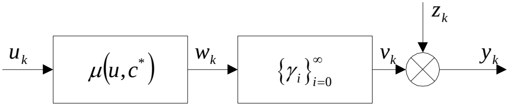
finite variance / finite 4th order moment

ergodicity

Identification methods

- **parametric**
  - least squares
  - instrumental variables
- **non-parametric**
  - kernel regression
  - orthogonal expansion

Two-stage identification of Hammerstein systemu



Hammerstein system

$$R(u) = \gamma_0 \mu(u) + \zeta$$

$$\widehat{w}_k = \widehat{R}(u_k) - \widehat{R}(0)$$

$$\{(u_k, \widehat{w}_k)\}_{k=1}^{N} \qquad \{(\widehat{w}_k, y_k)\}_{k=1}^{N}$$

Parameter estimation of static nonlinear block

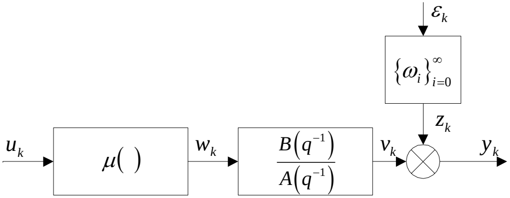
Stage 1 (non-parametric): Using observations $\{(u_k, y_k)\}_{k=1}^{M}$, for N_0 fixed points $\{\overline{u}_n;\ n = 1, 2, ..., N_0\}$ estimate $\{w_n = \mu(\overline{u}_n, c^*);\ n = 1, 2, ..., N_0\}$

$$\widehat{w}_{k,M} = \frac{\sum_{k=1}^{M} y_k K\left(\frac{u-u_k}{h(M)}\right)}{\sum_{k=1}^{M} K\left(\frac{u-u_k}{h(M)}\right)}, \qquad \text{or} \qquad \widehat{w}_{k,M} = \frac{\sum_{i=0}^{q(M)} \widehat{\alpha}_{i,M}\varphi_i(u)}{\sum_{i=0}^{q(M)} \widehat{\beta}_{i,M}\varphi_i(u)}$$

Stage 2 (parametric): Minimize the least squares criterion

$$\widehat{Q}_{N_0,M}(c) = \sum_{n=1}^{N_0} \left[\widehat{w}_{n,M} - \mu(\overline{u}_n, c)\right]^2 \to \min_c$$

Identification of ARMA block



Hammerstein system with ARMA block

$$
\begin{aligned}
v_k &= b_0 w_k + \ldots + b_s w_{k-s} + a_1 v_{k-1} + \ldots . + a_p v_{k-p} \\
\theta &= (b_0, b_1, \ldots, b_s, a_1, a_2, \ldots, a_p)^T \\
\vartheta_k &= (w_k, w_{k-1}, \ldots, w_{k-s}, y_{k-1}, y_{k-2}, \ldots, y_{k-p})^T \\
y_k &= \vartheta_k^T \theta + \bar{z}_k, \qquad \bar{z}_k = z_k - a_1 z_{k-1} - \ldots - a_p z_{k-p} \\
Y_N &= \Theta_N \theta + Z_N, \quad \Theta_N = (\vartheta_1, \ldots, \vartheta_N)^T, \quad Z_N = (\bar{z}_1, \ldots, \bar{z}_N)^T
\end{aligned}
$$

Nonparametric instrumental variables

$$\widehat{\theta}^{(IV)}_{N,M} = (\widehat{\Psi}^T_{N,M}\widehat{\Theta}_{N,M})^{-1}\widehat{\Psi}^T_{N,M} Y_N$$

where

$$\begin{aligned}
\widehat{\Theta}_{N,M} &= (\widehat{\vartheta}_{1,M}, \ldots, \widehat{\vartheta}_{N,M})^T \\
\widehat{\vartheta}_{k,M} &= (\widehat{w}_{k,M}, \ldots, \widehat{w}_{k-s,M}, y_{k-1}, \ldots, y_{k-p})^T \\
\widehat{\Psi}_{N,M} &= (\widehat{\psi}_{1,M}, \ldots, \widehat{\psi}_{N,M})^T \\
\widehat{\psi}_{k,M} &= (\widehat{w}_{k,M}, \ldots, \widehat{w}_{k-s,M}, \widehat{w}_{k-s-1,M}, \ldots, \widehat{w}_{k-s-p,M})^T
\end{aligned}$$

Limit properties

Static nonlinear block

$$\widehat{R}_M(\overline{u}_n) = R(\overline{u}_n) + O(M^{-\tau}) \implies \widehat{c}_{N_0,M} = c^* + O(M^{-\tau})$$

Linear dynamic block

$$\left|\widehat{R}_M(u) - R(u)\right| = O(M^{-\tau})$$
$$NM^{-\tau} \to 0, \qquad M \sim N^{(1+\alpha)/\tau}$$
$$\implies \left\|\widehat{\theta}^{(IV)}_{N,M} - \theta\right\| = O(N^{-\min(\frac{1}{2},\alpha)})$$

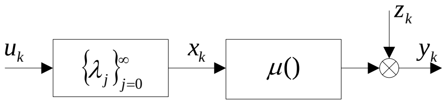
Identification under short measurement sequence

Semi-parametric approach

$$\hat{\mu}_1(u) = \hat{p}(u) + \frac{\sum_{k=1}^{N} (y_k - \hat{p}(u))\, K\left(\frac{u-u_k}{h_N}\right)}{\sum_{k=1}^{N} K\left(\frac{u-u_k}{h_N}\right)}$$

$$\begin{aligned} \hat{\mu}_2(u) &= \lambda_N \hat{p}(u) + (1-\lambda_N)\,\hat{\mu}(u) \\ \lambda_N &\to 0, \text{ as } N \to \infty \end{aligned}$$

A censored sample mean approach



Wiener system

$$\delta_k(x) \triangleq \sum_{j=0}^{k-1} |u_{k-j} - x| \lambda^j$$

$$\widehat{\mu}_M(x) = \frac{\sum_{k=1}^{M} y_k \cdot K\left(\frac{\delta_k(x)}{h(M)}\right)}{\sum_{k=1}^{M} K\left(\frac{\delta_k(x)}{h(M)}\right)}$$

Consistency conditions

Theorem

If $h(M) = d(M)\log_\lambda d(M)$, *as* $d(M) = M^{-\gamma(M)}$ *and* $\gamma(M) = \left(\log_{1/\lambda} M\right)^{-w}$, *then for each* $w \in \left(\frac{1}{2}, 1\right)$ *it holds that*

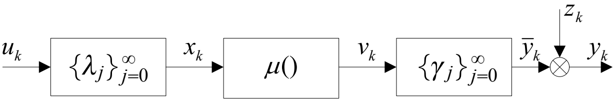
$$\lim_{M\to\infty} E\left(\widehat{\mu}_M(x) - \mu(x)\right)^2 = 0$$

Two-stage parameter estimation

$$Q_{N_0,M}(c) = \sum_{i=1}^{N_0} \left(\widehat{\mu}_M(x^{(i)}) - \mu(x^{(i)}, c) \right)^2$$

$$\widehat{c}_{N_0,M} = \arg\min_c Q_{N_0,M}(c)$$

Wiener-Hammerstein system



Wiener-Hammerstein (L-N-L sandwich) system

$$\widehat{\mu}_N^{(1)}(x) = \frac{\sum_{k=1}^{N} y_k \cdot K\left(\frac{\sum_{j=0}^{k} |u_{k-j} - x| \lambda^j}{h(N)}\right)}{\sum_{k=1}^{N} K\left(\frac{\sum_{j=0}^{k} |u_{k-j} - x| \lambda^j}{h(N)}\right)}$$

$$\widehat{\mu}_N^{(2)}(x) = \frac{\sum_{k=1}^{N} y_k \prod_{i=0}^{p} K\left(\frac{x - u_{k-i}}{h(N)}\right)}{\sum_{k=1}^{N} \prod_{i=0}^{p} K\left(\frac{x - u_{k-i}}{h(N)}\right)}$$

Identification of linear blocks

$$\hat{\varkappa} = \left(\sum_{k=1}^{N} \phi_k \phi_k^T K\left(\frac{\Delta_k}{\eta}\right) \right)^{-1} \left(\sum_{k=1}^{N} \phi_k y_k K\left(\frac{\Delta_k}{\eta}\right) \right)$$

where

$$\begin{aligned}
\{\varkappa_i\} &= \{\lambda_i\} * \{\gamma_i\} \\
\phi_k &= \left(u_k, u_{k-1}, \ldots, u_{k-(p+q)}\right)^T \\
\Delta_k &= \max_{j=0,1,\ldots,p+q} \left|u_{k-j}\right| \\
K\left(\frac{\Delta_k}{\eta}\right) &= \begin{cases} 1, \text{ as } \left|\Delta_k\right| \leq \eta \\ 0, \text{ as } \left|\Delta_k\right| > \eta \end{cases}
\end{aligned}$$

Limit properties

Theorem

If $\eta \sim N^{-\alpha}$, *where* $\alpha \in \left(0, \frac{1}{p+q+3}\right)$ *then*

$$\widehat{\varkappa}_\tau \to \varkappa_\tau, \qquad \tau = 0, 1, ..., p+q$$

with probability 1 *as* $N \to \infty$, *where* $\varkappa_\tau = \lambda_\tau * \gamma_\tau$.

Summary

Advanages of hybrid/combined methods

- identification under proor or partial prior knowledge
- allow to support selection of structure
- consistency proved analytically
- reduction of variance
- generalization, mild assumptions