$$X_N = \begin{bmatrix} x_1^T \\ \vdots \\ x_N^T \end{bmatrix} \qquad Y_N = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}$$

$P_{N+1} = P_N + \text{poprawka}$

$$\hat{a} = \underbrace{(X_N^T X_N)^{-1}}_{P_N} X_N^T Y_N$$

off-line

$N \geqslant s$

warunek konieczny niewystarczający

$$X_N^T X_N = \overbrace{\begin{bmatrix} \uparrow & \uparrow & & \uparrow & & \uparrow \\ x_1 & x_2 & \cdots & x_k & \cdots & x_N \\ \downarrow & \downarrow & & \downarrow & & \downarrow \end{bmatrix}}^{X_N^T} \cdot \begin{bmatrix} \leftarrow x_1^T \rightarrow \\ \leftarrow x_2^T \rightarrow \\ \vdots \\ \leftarrow x_k^T \rightarrow \\ \vdots \\ \leftarrow x_N^T \rightarrow \end{bmatrix} = [x_1 \cdot x_1^T] + x_2 x_2^T + \ldots + x_N x_N^T$$

$k = 1 \ldots N$

s

$$X_{N+1}^T X_{N+1} = \begin{bmatrix} \cdots & \cdots & x_{N+1} \end{bmatrix} \begin{bmatrix} \vdots \\ \leftarrow x_{N+1}^T \rightarrow \end{bmatrix} = \ldots + x_{N+1} x_{N+1}^T$$

$$X_{N+1}^T X_{N+1} = X_N^T X_N + x_{N+1} x_{N+1}^T$$

$$X_N^T Y_N = \begin{bmatrix} \uparrow & \uparrow & & \uparrow & & \uparrow \\ x_1 & x_2 & \cdots & x_k & \cdots & x_N \\ \downarrow & \downarrow & & \downarrow & & \downarrow \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \\ \vdots \\ y_N \end{bmatrix} = x_1 y_1 + x_2 y_2 + \ldots + x_N y_N = \sum_{k=1}^{N} x_k y_k$$

x_{N+1} $\qquad$ y_{n+1}

$$X_{N+1}^T Y_{N+1} = X_N^T Y_N + x_{N+1} \cdot y_{n+1}$$

$[\,\vdots\,] \cdot [\,\cdot\,]$