

# **NONPARAMETRIC APPROACH TO IDENTIFICATION OF BLOCK-ORIENTED SYSTEMS**

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# Statement of the problem

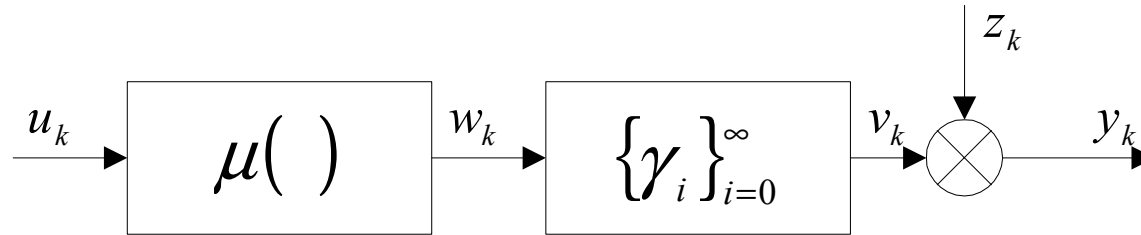


Fig.1. Hammerstein system

$$y_k = \sum_{i=0}^{\infty} \gamma_i \mu(u_{k-i}) + z_k$$

$$R(u) = \mathbf{E}\{y_k \mid u_k = u\} = c\mu(u) + s$$

# NONPARAMETRIC METHODS

## Kernel regression estimation

$$\hat{\mu}_M(u) = \frac{\sum_{i=1}^M y_i K\left(\frac{u - u_i}{h(M)}\right)}{\sum_{i=1}^M K\left(\frac{u - u_i}{h(M)}\right)}$$

*Theorem 1.* If  $h(M) \rightarrow 0$  and  $Mh(M) \rightarrow \infty$  as  $M \rightarrow \infty$  then

$$\hat{\mu}_M(u) \rightarrow \mu(u)$$

in probability as  $M \rightarrow \infty$  in each continuity point of  $\mu(\cdot)$  and the input probability density.

*Theorem 2.* If  $\mu(\cdot)$  is twice differentiable in the point  $u$  then for  $h(M) = O(M^{-1/5})$  it holds that

$$\left| \hat{\mu}_M(u) - \mu(u) \right| = O(M^{-2/5})$$

## Orthogonal expansion

$$\mu(u) = \frac{g(u)}{f(u)}, \text{ where } g(u) = \mu(u)f(u)$$

$$g(u) = \sum_{i=0}^{\infty} a_i \phi_i(u), \quad f(u) = \sum_{i=0}^{\infty} b_i \phi_i(u)$$

$$a_i = \mathbf{E} y_k \phi_i(u_k), \quad b_i = \mathbf{E} \phi_i(u_k)$$

$$\hat{a}_i = \frac{1}{M} \sum_{k=1}^M y_k \phi_i(u_k), \quad \hat{b}_i = \frac{1}{M} \sum_{k=1}^M \phi_i(u_k)$$

$$\hat{\mu}_M(u) = \frac{\sum_{i=0}^{q(M)} \hat{a}_i \phi_i(u)}{\sum_{i=0}^{q(M)} \hat{b}_i \phi_i(u)}$$

The convergence conditions:

trigonometric series  $\lim_{M \rightarrow \infty} \frac{q^2(M)}{M} = 0$

Legendre series  $\lim_{M \rightarrow \infty} \frac{q^2(M)}{M} = 0$

Laguerre series  $\lim_{M \rightarrow \infty} \frac{q^6(M)}{M} = 0$

Hermite series  $\lim_{M \rightarrow \infty} \frac{q^{5/3}(M)}{M} = 0$

Daubechies wavelets  $\lim_{M \rightarrow \infty} \frac{2^{2q(M)+2}}{M} = 0$

# COMBINED PARAMETRIC- NONPARAMETRIC ALGORITHMS



## Estimation of the static nonlinearity

$$\mu(u_k) = \mu(u_k, c) \qquad \mu(u_k) = e^{c_1 u} + \sin(c_2 u_k)$$

$$\hat{c}_{N,M} = \arg \min_c \sum_{k=1}^N \left( \hat{w}_{k,M} - \mu(u_k, c) \right)^2$$

$$\hat{c}_{N,M} = \left( \Phi_N^T \Phi_N \right)^{-1} \Phi_N^T \hat{W}_{N,M}$$

$$\Phi_N = (\phi_1(u_k), \phi_2(u_k), \dots, \phi_N(u_k))^T$$

$$\phi(u_k) = (f_1(u_k), f_2(u_k), \dots, f_m(u_k))^T$$

## Identification of the linear dynamics

$$y_k = \vartheta_k^T \theta + z_k$$

$$\theta = (\alpha_0, \alpha_1, \dots, \alpha_s, \beta_1, \beta_2, \dots, \beta_p)^T$$

$$\vartheta_k = (w_k, w_{k-1}, \dots, w_{k-s}, y_{k-1}, y_{k-2}, \dots, y_{k-p})^T$$

$$\hat{\theta}_{N,M} = \left( \hat{\Psi}_{N,M}^T \hat{\Theta}_{N,M} \right)^{-1} \hat{\Psi}_{N,M}^T Y_N$$

$$Y_N = (y_1, y_2, \dots, y_N)^T$$

$$\hat{\Theta}_{N,M} = (\vartheta_1, \vartheta_2, \dots, \vartheta_N)^T$$

# Nonparametric instrumental variables

$$\psi_k^* = (w_k, \dots, w_{k-s}, \bar{y}_{k-1}, \dots, \bar{y}_{k-p})^T \quad \psi_k^\# = (w_k, \dots, w_{k-s}, \tilde{y}_{k-1}, \dots, \tilde{y}_{k-p})^T$$

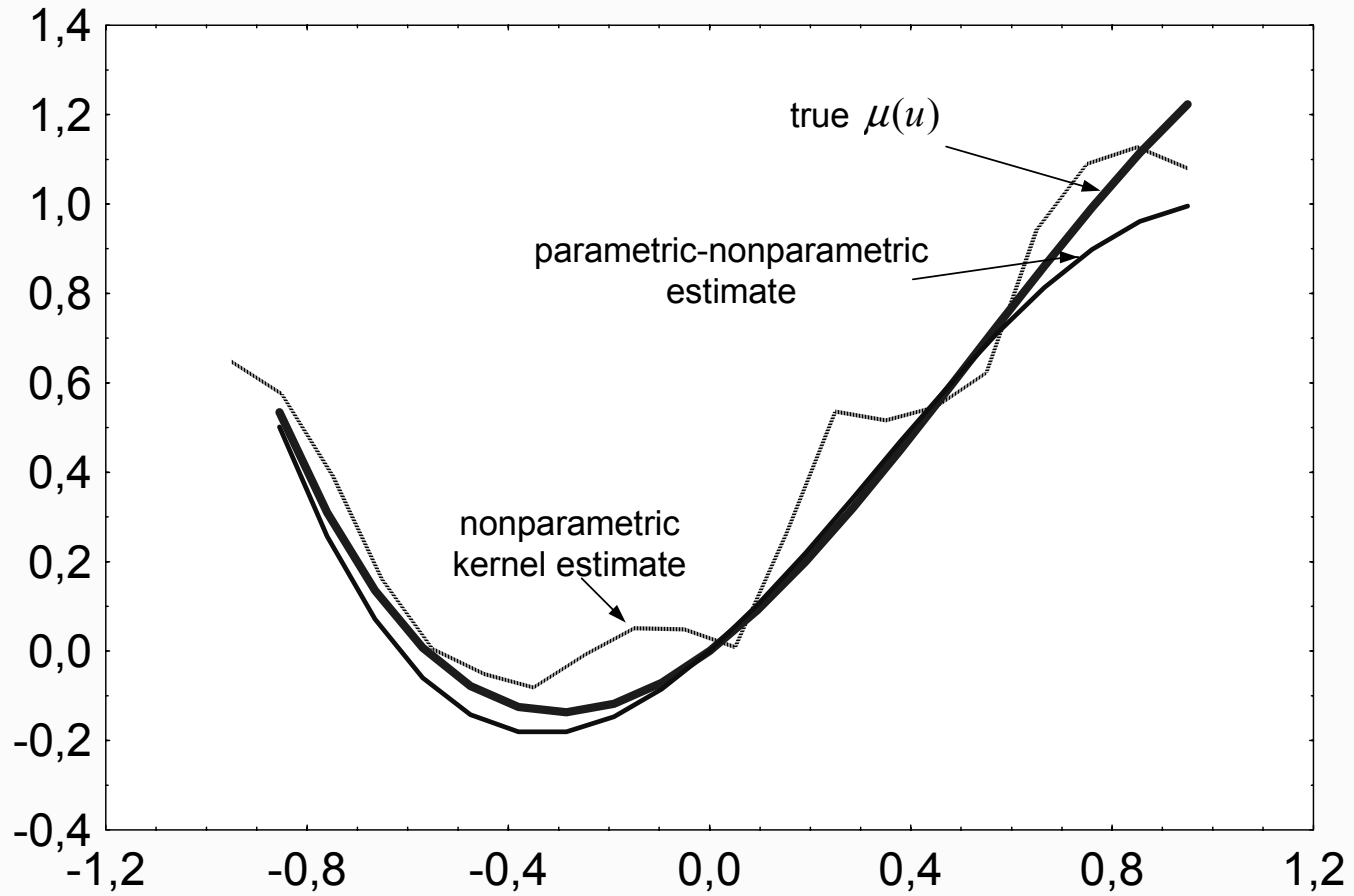
$$\tilde{y}_k = \sum_{i=0}^{APR} \hat{\gamma}_{i,M} \hat{w}_{k-i,M}$$

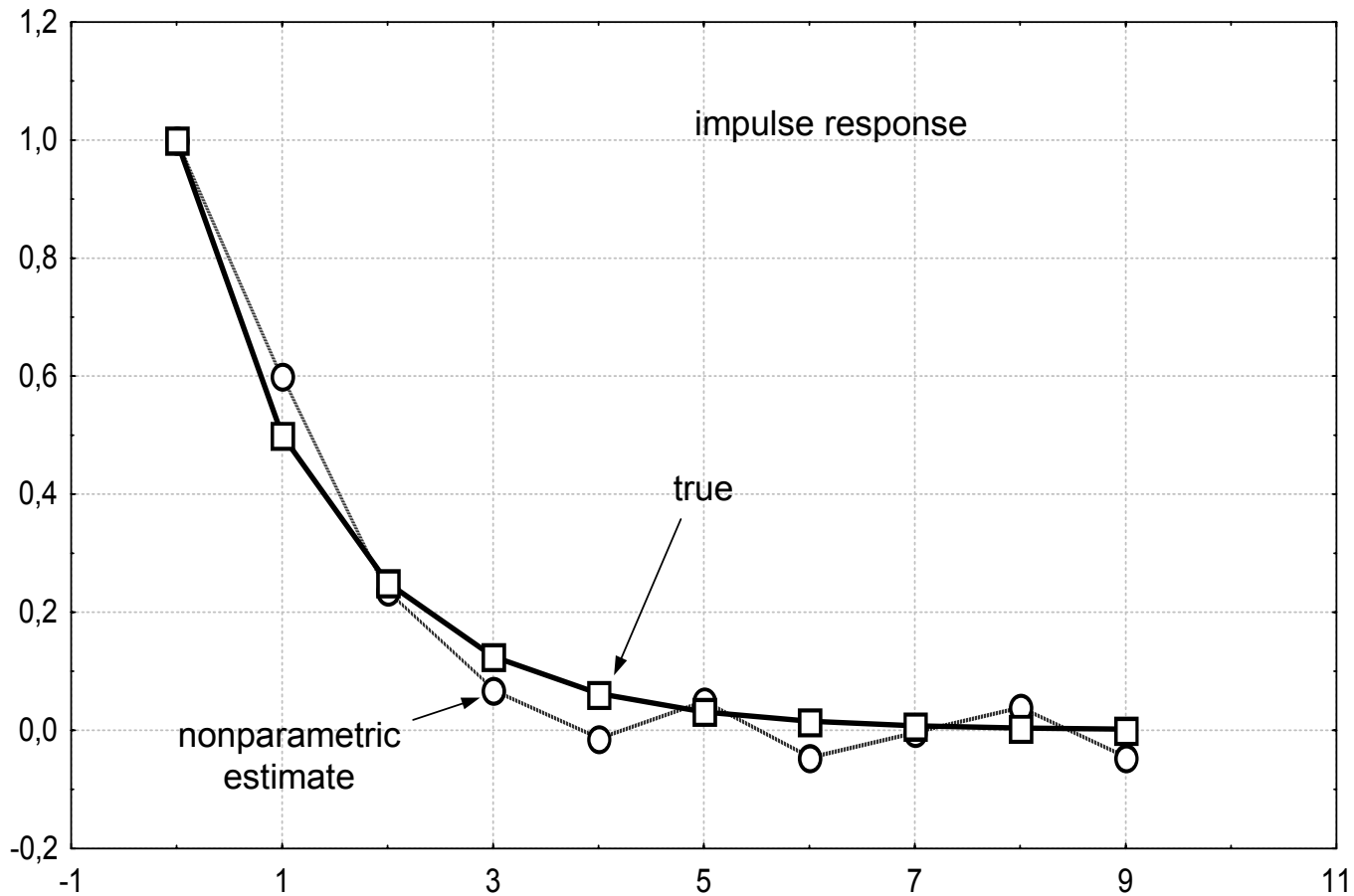
$$\hat{\gamma}_{i,M} = \hat{\chi}_{i,M} / \hat{\chi}_{0,M}$$

$$\hat{\chi}_{i,M} = \frac{1}{M} \sum_{k=1}^{M-i} (y_{k+i} - \bar{y})(u_k - \bar{u})$$

$$\bar{y} = \frac{1}{M} \sum_{k=1}^M y_k, \quad \bar{u} = \frac{1}{M} \sum_{k=1}^M u_k$$

# Example





# Conclusions

- Each part is identified separately
- The convergence is strictly proved  
Hasiewicz, Z. and G. Mzyk (2004). Combined parametric-nonparametric identification of Hammerstein systems. *IEEE Transactions on Automatic Control*, vol. 49, pp. 1370-1376.  
Hasiewicz, Z. and G. Mzyk (2006). Nonparametric instrumental variables for Hammerstein system identification. *IEEE Transactions on Automatic Control* (submitted to)
- The method works under existence of correlated random noise